

*An Apparent Periodicity in the Yield of Wheat for Eastern
England, 1885 to 1905.*

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In a paper contributed to the Hann celebration volume of the *Meteorologische Zeitschrift*, I endeavoured to account for a curious relation between the yields of wheat for successive years in the eastern district of England by referring the variation to periodic components. The relation referred to will be understood from an inspection of the table of yields on p. 75, or the diagram which illustrates it. The yields for the years 1896, 1897, and so on, taken in chronological order, apparently "compensate" respectively those for 1895, 1894, and so on, taken in reversed chronological order. Each pair of years at equal intervals, one before and one after 1895—6, gives a mean value approximating very closely to 30·8 bushels per acre, the average yield for the 20 years 1885 to 1904. This relation is further illustrated by the yield for 1905, which was not ascertained when the paper referred to was written (November, 1905). The data have been issued since then by the Board of Agriculture, and the yield for 1905 comes out at 32·0 bushels per acre, which "compensates" the yield for 1886, 29·2 bushels per acre, the mean being 30·6, surprisingly near to 30·8.

The work of the paper referred to was based upon the supposition that a reversal of the kind indicated points to the yield being represented by the combination of a number of simple harmonic components, each having a node, ascending or descending, at the epoch 1895—6. At first I did not suppose that the components belonged to a harmonic series, and I tried to evaluate them by eliminating components of specified period, two years, four years, etc., by numerical process.* The simplification produced by taking the mean of consecutive years, and thus eliminating the simple periodic term of two years' period was considerable, and, in my contribution to the "Hann" volume, I dealt with the curve thus simplified. When I had selected periodic terms to give the best representation I could make of the curve of two-year means, one of the terms had a period of 11 years, and I could draw no practical distinction between the others and the harmonic components of a fundamental period of 11 years. In the result it was shown that the average yield for two consecutive years'

* I owe this method, which is very useful in dealing with complex periodic variations of unknown periodicities, to the suggestion of Professor G. Chrystal, of Edinburgh.

yield of wheat in the eastern counties of England between 1885 and 1904 was represented with remarkable fidelity by the equation

$$y = 31 + 2.8 \sin \frac{2\pi}{11} n + 0.4 \sin \frac{4\pi}{11} n - 1.2 \sin \frac{6\pi}{11} n + 1.2 \sin \frac{8\pi}{11} n,$$

where n is the number of years, counting backwards or forwards, from the point representing the mean for 1895—6.

The process of taking means of consecutive years not only eliminates the oscillation of two years' period, but it alters the amplitude and phase of all other periodic oscillations. It is also to be noted that an apparent oscillation of two years' period may correspond with a combination of the fifth and sixth terms of the harmonic series with a fundamental period of 11 years. These two terms have periods of $11/5$ or 2.2 years, and $11/6$ or 1.83 years respectively.

I have now extended the calculation to get, from what I will call the ascertained components of the curve of two-year means, the corresponding components of the curve of original values.

It is a matter of simple trigonometry to show that if a_1, a_2, a_3, a_4 are the amplitudes of the first four components in the original curve, a_1', a_2', a_3', a_4' the amplitudes of the components of the same periods in the curve of means for two successive years, then

$$\begin{aligned} a_1 \cos \frac{1}{2} \frac{2\pi}{11} &= a_1', & a_2 \cos \frac{1}{2} \frac{4\pi}{11} &= a_2', \\ a_3 \cos \frac{1}{2} \frac{6\pi}{11} &= a_3', & a_4 \cos \frac{1}{2} \frac{8\pi}{11} &= a_4'. \end{aligned}$$

The phase of each component is altered by half a year, and thus the components remain concurrent in a common node if they are so in the original.

The equation to the first four components in the original curve is therefore

$$y_0 = 2.9 \sin \frac{2\pi}{11} n + 0.5 \sin \frac{4\pi}{11} n - 1.8 \sin \frac{6\pi}{11} n + 2.8 \sin \frac{8\pi}{11} n.$$

To get the values for successive years we have now to put n equal successively to $\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2} \dots$, on the positive and negative sides of the node, which in the actual curve is between 1895 and 1896.

These values were computed for each year and a curve of residuals plotted. It is an irregular curve, for the values are necessarily affected to some extent by accidental circumstances and the residual variations are not very large. In order to represent the residual curve I took two equal components, each having an amplitude of 1 bushel per acre for the fifth and

sixth terms of the series, and an ascending node at the epoch 1895—6. The final representation of the annual sequence of the yield of wheat for Eastern England is by a periodic variation of 11 years' period with its five harmonic components, as follows:—

Period.....	11	$\frac{11}{2}$	$\frac{11}{3}$	$\frac{11}{4}$	$\frac{11}{5}$	$\frac{11}{6}$	years.
Amplitude	2.9	0.5	-1.8	+2.8	+1.0	+1.0	bushels per acre.

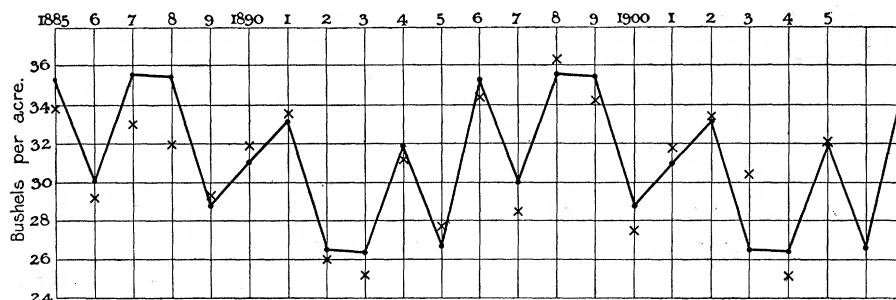
The + sign indicates that the component has an ascending node at the epoch 1895—6, the - sign a descending node. The values computed from this combination of components are shown in the table (p. 75) side by side with the actual values.

The agreement may fairly be called surprising; the differences are large for three years, 1887, 1888, and 1903, amounting in those cases to +2.7, +3.6, and -3.9 bushels respectively, but the error does not reach 2 bushels in any other year, and in 11 out of the 21 years it is less than a bushel.

When the amplitudes of the components were evaluated the yield for 1905 was unknown, the computed value was 31.9 bushels, the actual value turns out to be 32 bushels, thus showing a very striking agreement between calculation and actuality.

The relation between the computed values and the actual values is shown best by the diagram (fig. 1). It appears therefrom that the direction of the actual variation is the same as that of the computed variation for every step of consecutive years, though the amount of the variation is different to the extent indicated in what has been said above.

FIG. 1.—YIELD OF WHEAT, ENGLAND EAST, 1885—1905.



The line represents the combination of the harmonic components—

Period	11	$\frac{11}{2}$	$\frac{11}{3}$	$\frac{11}{4}$	$\frac{11}{5}$	$\frac{11}{6}$	years.
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Amplitude [31] +2.9 +0.5 -1.8 +2.8 +1.0 +1.0 bushels per acre.

Each has a node (+ascending, -descending) at 1895—6.

The crosses denote the actual yields of the several years.

The curious reversal in the variation of yield with reference to the epoch 1895—6 is thus satisfactorily accounted for by regarding the variation as the result of a simple periodic oscillation of 11 years' period with its five harmonic components of suitable amplitude.

Objection may be raised to the method adopted to obtain the amplitudes of the components. It is partly by numerical calculation and partly by trial with plotted residuals, and thus does not follow a strictly orthodox process. As a matter of fact, I was not prepared to find 11 years as the fundamental period, and the result has been extorted from me somewhat against my will by a long manipulation of the numbers having the reversal with reference to 1895—6 as the governing consideration. When I first took the matter up, although I considered the reversal to be obvious evidence of periodicity, I did not consider that I was entitled to assume any period as fundamental and I did not therefore regard the ordinary processes of harmonic analysis as applicable to this particular case.*

But at the time when my evaluation of the six components was completed I had the advantage of the assistance of Mr. H. E. Wood, Chief Assistant in the Transvaal Meteorological Service, who was working in the Meteorological Office for a period preparatory to his taking up his appointment in South Africa, and he was good enough to undertake the treatment of the figures for the yield of wheat on more orthodox methods.

He analysed harmonically the 11 years' period from 1891 to 1902 with the following results:—

Mean value ...	30·47	bushels.				
Period.....	11	$\frac{11}{2}$	$\frac{11}{3}$	$\frac{11}{4}$	$\frac{11}{5}$	$\frac{11}{6}$ years.
Amplitude	2·74	0·76	2·68	2·54	2·04	2·04 bushels.
Node (nearest						
year)	1896	1895	1896	1896	1895-6	1895-6
	(+)	(+)	(-)	(+)	(+)	(+)
	+ means ascending, - descending.					

It will be seen that except for the amplitudes of the fifth and sixth components the result is not very different from that which I obtained by a less regular process from a study of the values for the whole twenty years.

When the calculation derived from the analysis is extended to give

* The advantage of supposing that the components do not belong to a harmonic series is that any group of non-harmonic components, if persistent, must ultimately concur in a node, but the node and consequent reversal need not recur until after the lapse of what Professor Chrystal calls a "configuration period," which may be very long.

values for comparison with the whole series of 21 years, the errors are somewhat greater and the two-year oscillation is too conspicuous in the calculated values. The amplitude depends upon a single number in the harmonic computation and is therefore not capable of very accurate evaluation. The calculated value for 1905 is 32.8 bushels and is therefore a little further from actuality than the figure derived from my computation.

Mr. Wood also calculated the periodogram for the observations according to the method indicated by Professor Schuster. The periods dealt with were of 2, 3, 4 . . . up to 14 years and thereafter 16 years and 18 years. The method is from the nature of the case not very effective with data of such limited extent. Moreover, as there is only one wheat value for a whole year the components of 11 years cannot be well represented. Still the periodogram shows a prominence for a variation of an 11 years' period; $11/2$ is less marked, $11/3$ is very prominent, $11/4$ is just indicated.

One point in connection with the periodogram affords, as I understand it, a strong confirmation of the views represented in this paper. The ordinate representing the amplitude for the period of two years is *zero*. This result surprised me at first because the fluctuations from year to year are a most conspicuous feature of the line representing the actual values. My estimate of the amplitude of the fifth and sixth components which combine to give an oscillation of two years' period is 1 bushel; Mr. Wood's value, calculated from the data for 1891 to 1902, is 2 bushels. But the disappearance of the two-year oscillation from the periodogram deduced from 20 years' values is easily explained if the fluctuation is really the result of the "beat" of two components, one slightly greater and the other slightly less than two years. If the two components are equal in amplitude, the fluctuation, which is the combination of the two, changes its phase to the opposite in five and a-half years, so that the average fluctuation taken over 11 years should show no two-year oscillation at all; similarly, on the average of 22 years there should be no two-year oscillation, and the combination of the figures for 20 years, to give the periodogram ordinate for the two-year oscillation, shows that this inference is borne out by the actual numbers.

The periodogram, therefore, confirms the result that the two-year oscillation which is very conspicuous at some parts of the cycle is really a combination of two oscillations of nearly equal period which alters its phase within 20 years in such a way that on the average of the whole period there is no oscillation of two years' period at all, in other words, the even years are on the average neither better nor worse than the odd years.

The singular agreement between the actual figures and those derived from the periodic values for 11 years with its harmonic components may be attributed to chance by those who desire to adopt that explanation. It is certainly difficult to see how wheat can have much to do with variations which are periodic in a fraction of a year, even though the fraction be an improper one. But here again a curious point is disclosed. Those terms are most accentuated of which the periods come nearest to an exact number of years; the amplitudes and periods are (taking my own estimates) :—

Amplitudes.....	2.9	0.5	1.8	2.8	1.0	1.0.
Periods	11	5.5	3.67	2.75	2.2	1.83.

The least conspicuous component is that of five and a-half years, which has its maximum alternately in summer and winter, or other opposite periods of the year. The fourth component is stronger than the third; it is more nearly periodic in an exact number of years.

The fifth and sixth components together when they are in concurrence give a result greater than that of the third, and the period of the combined effect is of two years.

Thus there are subsidiary points which seem to discredit to a certain extent the doctrine of chance as an explanation of the fact from which the investigation started, namely, that the excesses and defects of the yield balance one another about the years 1895—6. Nor can I myself regard it as a matter of chance that a law of sequence derived from observations of a number of years continues to hold for extrapolations both in the past and in the future. I cannot see any other conclusion consonant with the facts but this—that during the 21 years—1885 to 1905—the yield of wheat in Eastern England has been very nearly identical with the sequence given by a periodic variation of 11 years and its five harmonic components. When one considers the vicissitudes to which the wheat crop is exposed, before the yield is estimated for the Board of Agriculture, it must be regarded as very remarkable not that there are exceptions to the law of sequence, but that there are so few of any importance.

Whether the law of sequence will continue to be that indicated by the composition of the curve for 21 years is another matter. The fundamental period may not be exactly 11 years; the components may prove to be after all not exactly the harmonics of the 11-year fundamental. Presumably we must wait until additional data have accumulated. It does not follow that the law, if true for the particular district I have chosen, the “England East” of the Meteorological Office, is necessarily true for every

county or every field. The data for two years taken at random indicate that the counties are differently affected in different years:—

Year.	Bedford.	Cambridge.	Essex.	Hertford.	Huntingdon.	Norfolk.	Suffolk.	England, East.
1903.....	29·5	31·4	28·8	28·2	30·5	33·1	29·4	30·4
1902.....	32·1	33·1	34·4	31·9	33·1	34·3	32·9	33·4

On that account I have not attempted to trace the relationship in the data compiled for different localities for longer periods.

A variation of about 11 years' period has been suggested for various solar and terrestrial phenomena, but I know of no case in which the sequence of phenomena is so completely accounted for in magnitude and direction as in the case of the yield of wheat for Eastern England. If it is not a matter of chance, and that seems too improbable, it must be a matter of considerable importance.

Table : Yield of Wheat for Eastern England.

1	2	3	4	5	6	7
Year.	Previous autumn rainfall in inches.	Yield computed from previous autumn rainfall (p. 70).	Actual yield in bushels per acre.	Yield computed from six harmonic components (p. 71).	Yield computed from the analysis of the yields 1891-1902 (Mr. Wood, p. 72).	Difference between calculated (column 5) and actual yield (column 4).
1885	7·30	29·9	33·8	35·3	36·2	+1·5
1886	12·63	18·2	29·2	30·1	26·6	+0·9
1887	6·92	30·8	32·9	35·6	37·7	+2·7
1888	6·67	31·3	31·9	35·5	37·6	+3·6
1889	5·37	34·2	29·3	28·8	28·1	-0·5
1890	6·90	30·8	31·8	31·0	31·7	-0·8
1891	5·60	33·7	33·5	33·2	32·9	-0·3
1892	8·22	27·9	26·0	26·5	26·9	+0·5
1893	10·19	23·6	25·2	26·4	23·7	+1·2
1894	6·86	30·9	31·1	31·9	32·8	+0·8
1895	7·90	28·6	27·7	26·7	25·6	-1·0
1896	6·74	31·2	34·4	35·3	36·2	+0·9
1897	9·49	25·1	28·5	30·1	26·6	+1·6
1898	5·14	34·7	36·3	35·6	37·7	-0·7
1899	5·23	35·1	34·2	35·5	37·6	+1·3
1900	7·83	28·8	27·5	28·8	28·1	+1·3
1901	4·83	35·4	31·8	31·0	31·7	-0·8
1902	4·78	35·5	33·4	33·2	32·9	-0·2
1903	4·26	36·6	30·4	26·5	26·9	-3·9
1904	8·96	26·3	25·2	26·4	23·7	+1·2
1905	3·80	37·6	32·0	31·9	32·8	-0·1

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This study arose out of the consideration of the relation between the autumn rainfall and the yield of wheat of the subsequent year. The relation as concerning the yield of wheat for Great Britain was the subject of a preliminary note to the Royal Society in February, 1905. The corresponding relation for the East of England is—

$$W=46-2\cdot2 R,$$

where W is the yield in bushels per acre and R the rainfall of the previous autumn. The figures from which this relation is computed are given in the table on the previous page, and a column of values derived from the formula is also given.

It is clear that the rainfall is subject to no such rigorous law of sequence as the wheat crop. A relation between the autumn rainfall and the wheat crop is sufficiently manifest, but evidently the fall of rain is subject to disturbances of an irregular character which have little counterpart in the curve of wheat values.
